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Context-invariant quasi hidden variable (qHV) modelling of all joint von Neumann measurements for an arbitrary Hilbert space

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We prove the existence for each Hilbert space of the two new quasi hidden variable (qHV) models, *statistically noncontextual and context-invariant,* reproducing all the von Neumann joint probabilities via non-negative values of real-valued measures and all the quantum product expectations—via the qHV (classical-like) average of the product of the corresponding random variables. In a context-invariant model, a quantum observable X can be represented by a variety of random variables satisfying the functional condition required in quantum foundations but each of these random variables equivalently models X under all joint von Neumann measurements, regardless of their contexts. The proved existence of this model *negates the general opinion* that, in terms of random variables, the Hilbert space description of all the joint von Neumann measurements for $\dim\mathcal{H} \geq 3$ can be reproduced only contextually. The existence of a statistically noncontextual qHV model, in particular, implies that *every N-partite quantum state admits a local quasi hidden variable model* introduced in Loubenets [J. Math. Phys. **53**, 022201 (2012)]. The new results of the present paper point also to the generality of the quasi-classical probability model proposed in Loubenets [J. Phys. A: Math. Theor. **45**, 185306 (2012)]. © *2015 AIP Publishing LLC.* [http://dx.doi.org/10.1063/1.4913864]

I. INTRODUCTION

The relation between the quantum probability model and the classical probability model has been a point of intensive discussions ever since the seminal publications of von Neumann,[1] Kolmogorov,[2] and Einstein, Podolsky, and Rosen (EPR).[3] In the frame of the quantum formalism, the interpretation of von Neumann measurements in classical probability terms, that is, via random variables and probability measures on a measurable space[4] $(\Omega, \mathcal{F}_\Omega)$, is generally referred to as *a hidden variable (HV) model*, but a setting of a HV model depends essentially on its aim. Moreover, in the literature, the HV models are usually divided into noncontextual and contextual.

In a noncontextual model, each quantum observable X is represented on $(\Omega, \mathcal{F}_\Omega)$ *by only one* random variable f_X with values in the spectrum sp$X$ of this observable X.

In a contextual model, not only there are quantum observables $X_\gamma, \gamma \in \Upsilon$, each modelled by a variety of random variables on $(\Omega, \mathcal{F}_\Omega)$, but also—which of these random variables represents an observable X_γ under a joint von Neumann measurement depends specifically on a *context* of this measurement, i.e., on other compatible quantum observables measured jointly with X_γ.

In foundations of quantum theory, where a HV model aims to reproduce in classical probability terms the statistical properties of *all* quantum observables on a Hilbert space $\mathcal{H}$, the intention to mimic all the properties of quantum averages and quantum correlations was realized via some additional functional assumptions[1,5,6] on a correspondence between random variables on $(\Omega, \mathcal{F}_\Omega)$ and quantum observables on $\mathcal{H}$. Under these functional assumptions, for $\dim\mathcal{H} \geq 3$, there does not exist a noncontextual HV model reproducing the Hilbert space description of all the joint von Neumann measurements (for details, see Sec. II).

In quantum information theory, a HV model aims to reproduce *only* the probabilistic description of a quantum correlation scenario upon a state ρ on a Hilbert space $\mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$, so that a

